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THE USE AND CALIBRATION OF DISTANCE MEASURING EQUIPMENT FOR PRECISE MENSURATION OF DAMS

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This report is intended as a practical guide to land surveyors using distance measuring equipment (DME) and making precise measurements of movements in dams and other large structures. The report consists of two parts. First, DME calibration, error sources, atmospheric corrections and correct use. Second, special techniques of trilateration and data reduction for precise mensuration on dams.

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THE USE AND CALIBRATION OF DISTANCE MEASURING EQUIPMENT FOR PRECISE MENSURATION OF DAMS

INTRODUCTION. The Corps of Engineers needs precise measurement of displacements within large engineering structures such as locks and dams. The regulations state that "Civil Works structures whose failure or partial failure would endanger the lives of the public or cause substantial property damage will be continuously evaluated to insure their structural safety and stability, and operational adequacy. Such evaluations, based upon period inspections supported when appropriate by programs of instrumentation, will be conducted to detect conditions of significant structural distress or operational inadequacy and to provide a basis for timely initiation of restorative and remedial measures."[1]

One of the most effective programs of instrumentation is high precision survey. This method provides a direct measure of displacement as a function of time, is reliable, and has fewer problems of evaluation than most other types of instrumentation. It is the purpose of this manual to provide assistance to the surveyor faced with the problem of making these precise surveys in a timely and cost-effective manner. The manual addresses itself to one particular method of precise survey, trilateration, and is not intended to provide instruction in general surveying, although some of the techniques discussed do apply.

Without question the most important recent advance in surveying has been the introduction of modern optical distance measuring instruments. In order to meet the needs of the surveyor these instruments come in a bewildering variety in terms of cost, range, accuracy, and ease of operation. When properly calibrated and used, distance measuring equipment (DME) is much more than simply a replacement for the steel tape. Instead, it provides new techniques, which may as yet be unfamiliar, to the surveyor, provides substantial savings in time and manpower, and leads to higher accuracies.

Unfortunately, DME also provides new problems, which must be dealt with. For example, the instruments must be calibrated together with their reflectors, error sources must be examined, and the proper use of temperature and pressure monitoring equipment becomes essential. A further purpose of this manual then is to deal with the problems of instrument selection, calibration, and use, particularly where measurements of high accuracy are required.

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[1]"Periodic Inspection and Continuing Evaluation of Completed Civil Works Structures," U. S. Army, Corps of Engineers, Regulation No. 1110-2-100, 26 Feb 1973.

PART I.

THE CALIBRATION AND USE OF DISTANCE MEASURING EQUIPMENT

DME OPERATION. All modern DME measure distance by timing, in an indirect fashion, how long it takes light to make a round trip to a reflector. By knowing the velocity of light, the distance may be calculated from $2d = vt$, where d is the distance to the reflector, v is the velocity of light, and t is the time required for light to travel to the reflector and back. Light travels 1 foot in approximately a nanosecond (1×10^{-9} seconds). If it were possible to time directly a pulse of light as it traveled to a target and back, the number of nanoseconds divided by 2 would be roughly the number of feet to the target. This is the principle of the laser range finder used by the Army. For the purpose of artillery, an accuracy of a few feet is sufficient, and direct timing of pulses provides the desired result. However for surveying, where accuracy requirements are much greater, another technique must be used to avoid the problems of timing a pulse to a small fraction of a nanosecond. In surveying a continuously operating source of light is used, and this light is modulated in a known way. In the case of DME, modulation simply means that the light is turned off and on in a regular fashion, usually sinusoidally. In figure 1a, a DME is shown emitting a modulated beam of light. The sinewave represents the amplitude of the light along the path of the beam at an instant of time. The modulation wavelength λ, not to be confused with the natural wavelength of the light itself, is determined by the rate at which the light is modulated and by the velocity at which it is traveling; $\lambda = v/f$, where λ is the modulation wavelength, v is the velocity at which the light is traveling, and f is the frequency at which the light is being modulated. In figure 1b, the light has been reflected from a mirror or corner cube reflector and has been returned to the instrument. A comparison is made in terms of phase between the light proceeding toward the reflector and that returning. In figure 1b, the reflector has been placed so that it is exactly a whole number of modulation wavelengths away from the instrument. The returning light is then found to be in phase with the outgoing light, and the operator knows that the distance to the reflector is a whole number times one-half the modulation wavelength of the instrument. In figure 1c, the reflector has been moved one-quarter wavelength farther away from the instrument. The returning beam is now one-half wavelength or 180° out of phase with the outgoing beam. The operator knows the distance to the reflector is now a whole number times one-half the modulation wavelength, plus a quarter of a modulation wavelength. Finally in figure 1d, the reflector has again been moved one-quarter wavelength farther away, and again the beam returns in phase with the outgoing beam. Note that in moving the reflector by one-half wavelength, the phase change has been a

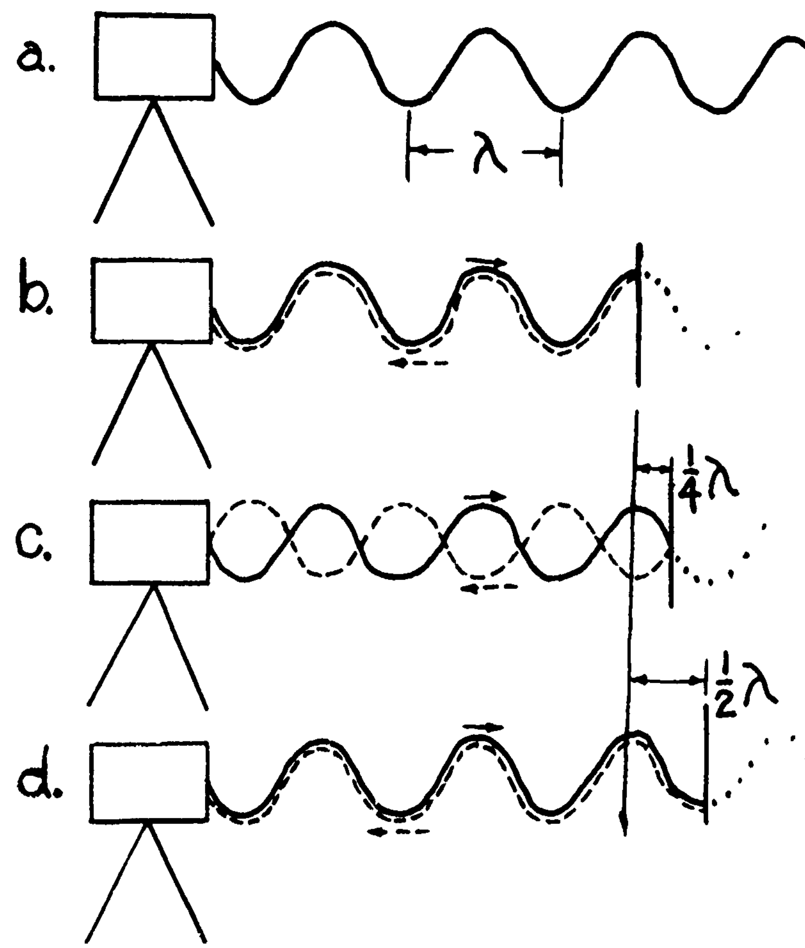


Figure 1. Phase Comparison

complete 360°. This also means that the same answer will be obtained every one-half wavelength so that additional modulation wavelengths are required to resolve ambiguities. As an example of how phase comparison techniques improve the resolution of DME, a typical instrument might be built with a modulation wavelength of 36 feet. With this instrument, a change in distance to the reflector of 18 feet corresponds to a phase change of 360°. A phase measurement may be measured to better than 1°, so that the resolution of the instrument becomes 18/360 = 0.05 feet. Certain instruments have resolutions as fine as 0.001 feet using the phase comparison technique.

This in brief, is how the modern DME works. It requires a knowledge of the velocity of light and uses modulation of the light at a precisely known frequency and phase comparison to obtain the distance. Each of these factors, however, introduces an error into the measurement. The next section deals with some of these errors.

DME ERROR SOURCES. This section refers specifically to error sources which are an inherent property of the instrument itself and are an addition to external factors, such as plumbing and measurements of refractive index. Most manufacturers of DME list the error of their instruments as (a) mm + (b) mm/km, where (a) and (b) are maximum values for a particular instrument model and manufacturer.

In the (a) portion of the error, several small errors are lumped together, which are independent of the length of the line being measured. The more important of these are

1. Instrument resolution.
2. Cyclic or delay line error.
3. Instrument-reflector calibration.
4. Offset.
5. Pointing.

The (b) portion of the error is due to the short- and long-term variations in the frequency standard used to control the modulation frequency. This is usually a quartz crystal, which may or may not be mounted within a small oven for temperature control.

While the errors described in (a) are generally grouped together, it is helpful to examine them individually, as occasionally special techniques may be used to lower the magnitude of certain of them.

Instrument resolution. Resolution is a property of the instrument that results from its original design. In the case of DME, it might be defined as the smallest change in distance to a target that causes a corresponding change in the reading obtained from the instrument. The resolution can be no finer than the scale or digital display can be read. On the other hand, a display which

can be read to a millimeter is no assurance that the resolution of the instrument is also a millimeter.

The simplest test for resolution is to mount a reflector so that it may be moved back and forth along a line connecting the reflector and the DME. Movement of a foot or two is sufficient. Affix a scale next to the reflector so that the distance the reflector is to be moved may be accurately measured. Make measurements with the reflector at five different positions along the scale. The calculation of resolution may then be performed as shown in table 1.

Table 1. DME Resolution Test

Reflector Position (inches)	(meters)	DME Measurement (meters)		Difference (meters)
1.5	0.0381	132.560		132.5219
6.0	0.1524	132.668		132.5156
9.0	0.2286	132.749		132.5204
15.5	0.3937	132.916		132.5223
20.0	0.5080	133.019		132.5110
23.5	0.5969	133.110		132.5131
			Mean	132.5174

The standard deviation σ = 0.0048 meters.

Resolution 4.8 mm.

In this example, the DME was positioned some 132 meters from the scale and reflector. The reflector was positioned at 1.5, 6.0, 9.0, 15.5, 20.0, and 23.5 inches according to the scale (column 1 of the table). A measurement of the distance was made with the DME for each position of the reflector (column 2). The values in column 1 are then subtracted from the corresponding values in column 2 to obtain the differences (column 3). Finally, the standard deviation is taken of the values in column 3, and the result, 4.8 mm, is the approximate resolution of the instrument.

<u>Cyclic or delay line error</u>. If the manner in which the light is modulated distorts the sinusoidal pattern of the outgoing beam or if the phase comparison technique of measuring the returning beam is less than perfect, a cyclic error will occur. The error is named cyclic because it repeats itself every modulation wavelength. If the effective modulation wavelength of a particular instrument is 10 feet and the cyclic error for a measurement of 5 feet is 0.02 feet, then the cyclic error at 15, 25, 35 feet would also be 0.02 feet. At the same time, the instrument might have zero cyclic error at 1.5, 11.5, 21.5 feet. A determination of cyclic error consists of making comparative measurements throughout a full modulation wavelength.

From the manufacturers specifications, choose the highest modulation frequency (This will usually be between 10 megahertz and 80 megahertz). The effective modulation wavelength will be $\lambda = 3 \times 10^8/2f$ meters, where λ is the modulation wavelength, and f is the highest modulation frequency. For an instrument operating at a frequency of 15 megahertz, the effective modulation wavelength would be $\lambda = 3 \times 10^8/2(15 \times 10^6) =$ 10 meters. Divide this distance into a least 10 equal parts. For the 10-meter wavelength, a convenient value might be 13 increments of 0.8 meters each.

Make measurements with the DME of the distance to a reflector as it is moved in increments of length along a straight line over an entire modulation wavelength. Record the data in a manner similar to that shown in table 2. The measurements (table 2) contain

Table 2. Cyclic or Delay Line Error

Increment (meters)	Measurement Distance (meters)	Difference (2) - (1) (meters)
0	80.503	80.503
.8	81.308	80.508
1.6	82.111	80.511
2.4	82.915	80.515
3.2	83.710	80.510
4.0	84.507	80.507
4.8	85.303	80.503
5.6	86.100	80.500
6.4	86.896	80.496
7.2	87.692	80.492
8.0	88.495	80.495
8.8	89.297	80.497
9.6	90.101	80.501
10.4	90.904	80.504

Standard deviation $\sigma = 6.7$ mm.

cyclic or delay line errors but also include resolution errors as found in the previous section. When dealing with errors of this type, the simple difference between the two cannot be taken. If x and y are errors of different types, the sum is given by $z^2 = x^2 + y^2$. In table 2, the cyclic error plus the resolution error alone was 4.8 mm. The difference, $y^2 = (6.7)^2 - (4.8)^2$, is the cyclic error $y = 4.7$ mm.

Instrument-reflector calibration. When a DME is received from the manufacturer, one or more reflectors are usually received at the same time and these have been assigned a constant by the manufacturer. This constant is to be added or subtracted from the

distance reading in order to obtain a correct distance. In most cases, the constant is sufficiently accurate for routine work. However, for greater precision or for reflectors that are obtained from other sources, it is necessary to determine accurately the constant of each reflector. Figure 2 shows two reflectors, both of which are mounted at the same distance from the measuring instrument. Because the reflectors are at different positions within their mounts, they give different readings for the same distance. The difference between the true distance and the reading obtained with a particular reflector is the reflector constant. The constant may be determined by following these steps. Lay off three points along a straight line. Mark the second point about 100 meters from the first. Determine the shortest modulation wavelength for the instrument, and made the distance between the second and third points approximately a whole number of modulation wavelengths (±10 millimeters). Choose a distance between the second and third point of about 100 meters to avoid refraction errors. Set the DME on the first point, and measure to the reflector plumbed over the second point. Next, measure to the <u>same</u> reflector plumbed above the third point. Both measurements will be in error by the amount of the reflector constant, but the difference of the two measurements will be the true length, free of this error. Move the instrument to the second point, and measure the distance to the <u>same</u> reflector mounted above the third point. The amount that must be added or subtracted from this measurement in order to obtain the true length is the reflector constant. An example of the determination of a reflector constant is given in table 3. The reflector constant is only good

Table 3. Determination of Reflector Constant

LINE	LENGTH (from DME)	
Point 1 to Point 2	113.406	
Point 1 to Point 3	183.409	
Difference	70.003	= true length
Point 2 to Point 3	70.036	= A (from DME)
True length	70.003	= B (difference)
Reflector constant	-.033	= (B - A)

0.033 must be subtracted from the instrument reading to correct for the reflector constant.

True distance

Point 1 to Point 2 113.406 - .033 = 113.373

Point 1 to Point 3 183.409 - .033 = 183.376.

for that particular instrument-reflector combination. If the reflector is used with another instrument, the constant will have to be redetermined for that instrument. Finally, the tests of

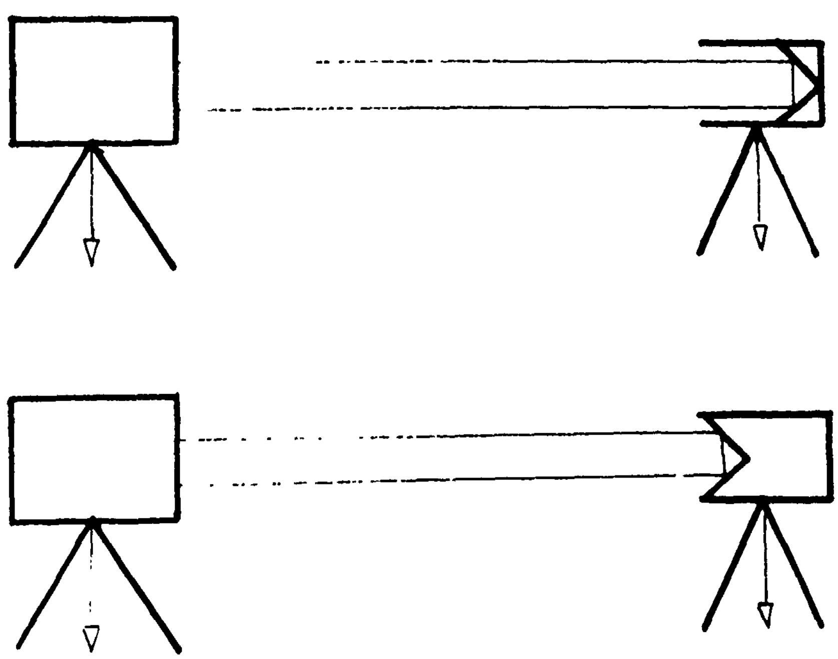


Figure 2. Reflector Calibration

reflector constant should be repeated several times with the difference from test to test being no larger than the resolution of the instrument.

If other reflectors are to be used with the DME they may be calibrated in the same manner, or one reflector may be selected as a standard. When this reflector has been calibrated with the instrument, it may be mounted at a distance of approximately 100 meters and used to measure the distance. After applying the reflector constant to the measured distance, the length is considered to be correct. Each of the other reflectors is then mounted in turn over the same point. Any difference in the measured distance is due to a difference in reflector constant. Table 4 gives an

Table 4. Calibration of Several Reflectors

Reflector	Measured Length	Constant
1	100.687	-0.031
2	100.680	-0.024
3	100.696	-0.040
4	100.582	+0.074
5	100.580	+0.076

Measurement of line with standard reflector	- 100.689
Reflector constant	- -.033
True line length	- 100.656

example of the calibration of several reflectors. The differences in reflector constants shown in the table can occur in actual practice, which shows the necessity for calibration of each reflector.

Offset. The reflector calibration includes two sources of error. The first error is caused by the reflector not being optically above the point over which it is plumbed. The second error is caused by the DME electrical or optical center, the point from which the instrument measures, not being above the point over which it is plumbed. However, both errors are corrected when the instrument reflector combination is calibrated. On the other hand, if the measuring center of the instrument should shift as the electronics age, the reflector constant would no longer compensate for this shift, and an offset error would result. Even so, it is possible to measure the magnitude of the offset error. At weekly intervals, simply measure a short line (100 meters) using the same reflector (the line should be outside so that the instrument will be subject to a variety of temperatures). If each measurement is made using the same procedures, the differences in length in excess of the resolution error are due to changes in the offset or

electrical center of the DME. Changes of 10 millimeters may occur in some instruments.

Pointing Error. The modulation wavefront issuing from a properly designed and operating instrument is at all points equidistant from the center of the instrument (figure 3). It might be likened to the waves around a stone dropped into water. Sometimes, however, the wavefront may be distorted in passing through the modulator, and then a portion of the wave may be ahead or behind the remainder. In figure 3, the instrument sees both reflectors as equidistant because the phase of the modulated wave is the same for both. If the instrument is moved in azimuth slightly, the distance that is read would change to the proper value.

This type of error may be detected simply by multiple pointings at a reflector. If different pointings yield different answers, it may be necessary to take several readings in the field, swinging off the target and then back until two or three sets of readings agree well. Practice in the field may help eliminate this problem as an experienced operator tends to point and adjust an instrument in the same way for each measurement.

Total Error. The previous sections have treated individually the various types of error in the (a) portion of the error which may be present in DME. It may be, however, that the total (a) error is all that is desired. In this case, one series of measurements will provide an indication of the total error. A line of five monuments is required as shown below.

The instrument is set over each point in turn, and the following distances are measured:

From A: AB, AC, AE
From B: BA, BC
From C: CA, CB, CD, CE
From D: DC, DE
From E: EA, EC, ED.

Use the same reflector for all measurements. If three prisms are required for longer lengths, one or two prisms may be masked off for the shorter measurements. Let AE be no longer than 750 meters and let no two segments be of the same length, or a multiple of the length, so that a good sampling of the modulation wavelength may be obtained. Apply refractive index corrections to the measurements, and attempt to measure all the lines during a single day. It is not

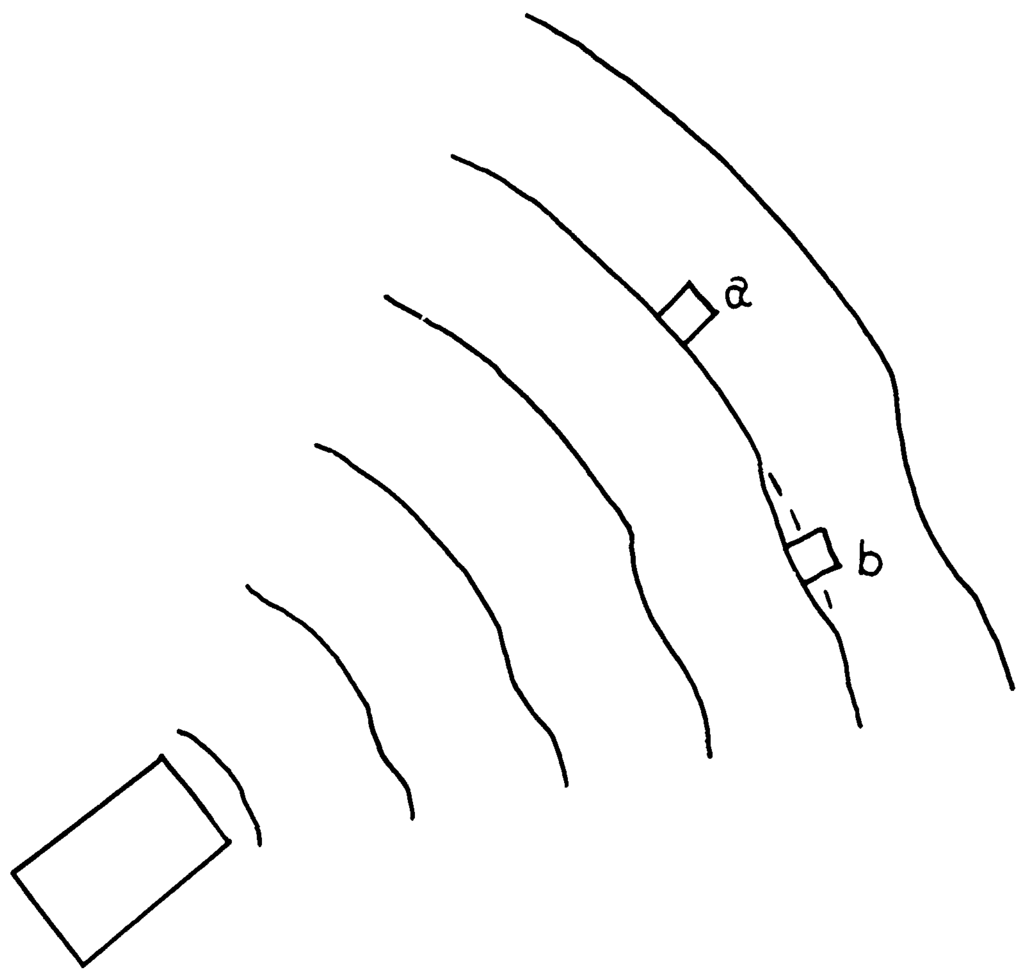


Figure 3. Pointing Error

necessary to know the true lengths of the segments. Calculate the total length, ℓ, of the line in the following ways:

$$\ell_1 = \frac{AB + BC + CD + DE + ED + DC + CB + BA}{2}$$

$$\ell_2 = \frac{AC + CE + EC + CA}{2}$$

$$\ell_3 = \frac{AE + EA}{2} \qquad (1)$$

The error in each of the measurements consists of a random portion owing to noise, resolution, and cyclic error and of a constant portion owing to offset and reflector calibration errors. The constant portion of the error is present with the same sign and approximately the same magnitude for each measurement. Thus, ℓ_1 contains this error 8/2 = 4 times, ℓ_2 contains the error 4/2 = 2 times, and ℓ_3 2/2 = 1 time. In table 5, the lengths measured on a

Table 5. Lengths of a Five Station Baseline

Segment	Measured Length (meters)	Length corrected for offset (meters)
AB	81.236	81.232
AC	176.926	176.922
AE	690.149	690.145
BA	81.241	81.237
BC	95.696	95.692
CA	176.929	176.925
CB	95.692	95.688
CD	274.815	274.811
CE	513.222	513.218
DC	274.816	274.812
DE	238.409	238.405
EA	690.146	690.142
EC	513.226	513.222
ED	238.414	238.410

ℓ_1 = 690.1595	ℓ_6 = 690.140
ℓ_2 = 690.1515	ℓ_7 = 690.147
ℓ_3 = 690.1475	ℓ_8 = 690.145
ℓ_4 = 690.140	ℓ_9 = 690.142
ℓ_5 = 690.147	σ .0033

five station line (column 2) and the lengths ℓ_1, ℓ_2, and ℓ_3 are listed. Let c be the constant portion of the instrument error. Then ℓ_3, which represents setting the instrument up once, will be the true length (neglecting the random portion of the error for the

moment) t, plus the constant error ($\ell_3 = t + c$).

Similarly $\ell_2 = t + 2c$ and $\ell_1 = t + 4c$ because the instrument was set up 2 and 4 times respectively to complete the entire line. Subtracting we have

$$\ell_2 - \ell_3 = c = 0.004 \qquad c = 0.004$$
$$\ell_1 - \ell_2 = 2c = 0.008 \qquad c = 0.004$$
$$\ell_1 - \ell_3 = 3c = 0.012 \qquad c = 0.004 \tag{2}$$

Each time the instrument is set up an error of 0.004 meters is added to the measured length. In column 3 of table 5, 0.004 meters has been subtracted from the lengths of column 2. The random portion of the error can now be estimated by calculating the length of the line from:

$$\ell_4 = AB + BC + CD + DE$$
$$\ell_5 = ED + DC + CB + BA$$
$$\ell_6 = AC + CE$$
$$\ell_7 = EC + CA$$
$$\ell_8 = AE$$
$$\ell_9 = EA.$$

These lengths are also given in table 5 and have a standard deviation of 0.003 meters. The measurements of the five station baseline has shown that the instrument error was composed of two parts; (1) a long term offset error of 0.004 meters, and (2) a short term error of 0.003 meters. The sum, $E = \sqrt{(0.004)^2 + (0.003)^2} =$ 0.005 meters.

This is the magnitude of the error which might be expected from measurements made in the field on a given day.

Frequency. The (b) portion of the error in DME consists of the short- and long-term variations in the frequency standard used to control the light modulator. The only proper method of checking this frequency is through the use of a properly calibrated electronic counter. Some manufacturers provide a connector on the instrument so that the frequency may be easily monitored.

When purchasing an instrument, insist on a means of checking frequency. Many of the newer instruments do not use ovens for their crystals and may have rather large errors, particularly under extremes of temperature. The error is proportional to the length measured so that at short distances, it may not be detectable. If the instrument is being used to measure a precise baseline, an electronic counter should be used to measure the modulation frequency before and after the measurement.

ATMOSPHERIC CORRECTIONS. The accuracy of measurements made with DME depends not only upon the instrument itself but also upon a knowledge of the velocity of light along the measuring path. Techniques to be discussed later reduce this dependence to a minimum in the case of measuring movements in large structures, but in order to make absolute measurements, it is vital to have an accurate knowledge of the refractive index of the air along the line being measured. The velocity of light, the refractive index of the air, and the instrument itself are related in the following manner.

In a vacuum, light of all wavelengths travels at the same velocity, which has been carefully measured and found to be, 299,792.5 km per sec or 983,571,200 ft per sec. When not in a vacuum, light travels at a lower velocity determined by the temperature and pressure of the air and the wavelength of the light. The velocity dependence on wavelength is:

$$n_g = 1 + \left(287.604 + \frac{4.8864}{\lambda^2} + \frac{0.068}{\lambda^4}\right) \cdot 10^{-6}$$

where n_g is the refractive index, and λ is the wavelength of the light expressed in micrometers for a temperature of 0°C and 760 mm of mercury (H_g). The subscript n_g refers to the fact that the light is modulated, which is the case for DME. The refractive index is simply a measure of how much light is slowed down in traveling through a medium other than a vacuum and is related to the vacuum velocity by $V = C/n_g$. The vacuum velocity is C, the velocity of light in air is V, n_g is the refractive index. Some representative refractive indices used with modern DME are

Wavelength (micrometers)	Source	Refractive Index
0.48	Xenon arc	1.0003101
0.63	HeNe laser	1.0003002
0.91	GaAs diode (infrared)	1.0002936

These refractive indices are at standard conditions. For any other conditions of temperature and pressure, a new refractive index may be found from

$$n_a = 1 + \left(\frac{n_g - 1}{1 + \frac{T}{273.2}}\right)\left(\frac{P}{760}\right) \tag{3}$$

where T is in °C, and P is in mm of H_g. At this point, the

instrument manufacturer selects a standard set of conditions, about which he designs an instrument. He might, for example, design the DME to give a correct result at 15°C and 760 mm Hg, with an infrared diode. In this case, n_a = 1.0002783. In addition, a measuring interval of 10 meters might be selected to provide conveniently the accuracy intended by the manufacturer. Because DME measures round trip distance, a 20-meter modulation length would be necessary to achieve the 10-meter measuring interval. The modulation length, in turn, is related to the velocity of light through the modulating frequency by $F = V/\lambda$, where F is the modulating frequency, V is the velocity of light under the chosen conditions, and λ is the modulation wavelength, in this case 20 meters. The velocity is the vacuum velocity of light divided by the refractive index, or 299,792.5/ 1.0002783 = 298,960.5 km per sec. Thus, the modulating frequency would be 298,960.5/.020 km = 14.948025 mHz, and this is the operating frequency of the crystal supplied with the DME.

For the surveyor, these calculations have already been made, and the instrument has been supplied with the proper crystal in place. For the further convenience of the surveyor, the atmospheric corrections are supplied in the form of a handy circular slide rule, tables, or a simple nomogram, which gives a parts-per-million correction depending on temperature and pressure. This correction may be either dialed into the instrument or applied directly to the measured distance.

Since the slide rules and nomograms are not really adequate for the most precise work, it would be better however to apply the refractive index equation as a part of the computation. The appropriate equation for the refractive index correction or the temperature and pressure for which the instrument is standardized should be obtained from the manufacturer.

MEASUREMENT OF TEMPERATURE AND PRESSURE. When absolute accuracy in measuring is required, such as when a baseline is laid out, temperture and pressure measurements play a vital part. To give an idea of the magnitude of errors arising from the incorrect application of temperature and pressure corrections, a change of 1°C will cause roughly a one-part-per-million change in the observed distance. A change in atmospheric pressure of 0.1 inches of Hg will also cause a one-part-per-million change in distance. Of the two, temperature is the most difficult to measure correctly.

Pressure measurements can be made accurately in the field with a good aneroid barometer, which may be read to 0.01 inches of Hg. However, the aneroid barometer must be checked frequently against a mercury barometer. This check should be made as often as possible, preferably once a day. In an emergency, the sea level pressure may be obtained from the weather bureau or a local airport. This

pressure must then be reduced to a known elevation at which the barometer is placed for calibration. The equation for the change in pressure with elevation is $P_h = P_0\ (1-0.0000068754\ h)^{5.2561}$ where P_h is the pressure at an elevation of h feet, and P_0 is the sea level pressure.

Pressure measurements should be made at both ends of the line and the mean of the two values used in the refractive index equation. If it is not possible to place barometers at both ends of the line, place the barometer at the instrument end, and use the elevations of the two ends together with the pressure measured at the instrument to calculate the pressure at the other end. The equation given above may once again be used with P_h being the unknown pressure at one end of the line, h the difference in elevation between the two ends, and P_0 the pressure at the instrument. The algebraic sign of h may be determined by remembering the higher elevation will have the lower pressure.

Temperature is much more difficult to measure properly. There are two reasons for this. First, the measuring equipment must be well shielded from the sun's radiation. One way of doing this is to enclose the thermometer in a reflective insulating shield. This, however, permits the heat to build up within the shield and thus, a small fan or some other means must be used to move air over the temperature sensing device so that the true air temperature is read. The second and more serious cause for error is that temperatures measured at the end points of a line near the ground are a poor indication of the true temperature along the line. Studies have shown that during the day temperatures near the ground are much warmer than those 100 feet above the ground. A 5 degree difference is not uncommon. At night the reverse is true, temperatures near the ground are cooler than those above. Unfortunately, many lines to be measured are more than 100 feet above the ground over most of their lengths. Consequently, measurements of temperature near the ground introduce serious temperature errors into the calculation of length. One means of reducing this error is to mount the temperature sensor approximately 15 to 30 feet above the ground. However, even this measure does not completely solve the problem. Thus, temperature measurements are one of the major sources of error in the accurate determination of distance.

In measuring dams or other large structures, the errors are less important because displacement values are needed, and therefore relative, rather than absolute, distances may be used. Special techniques for use on dams will be discussed later.

There are times, however, when an accurate measurement of distance is needed. This is the case for a baseline or where it is desired to give scale to a figure. For a baseline, choose a site

where the terrain will allow temperature measurements to be made at the average height of the line above the ground at least every 500 meters along the entire length of the line plus the end points. When calculating the temperature along the line, give the end point values a weight of one and the intermediate values a weight of two. Thus, for a 1,500 meter line

Position	Temperature x Weight
Instrument End	21.5 x 1 = 21.5
500 meters	24.0 x 2 = 48.0
1,000 meters	23.0 x 2 = 46.0
Reflector End	22.5 x 1 = 22.5
Sum	138.0

$$138.0/6 = \text{weighted mean temp} = 23.0C \tag{4}$$

Perform the measurements three times, about 4 hours before sunset, 1 hour before sunset, and 2 hours after sunset. If agreement between the three sets is satisfactory, the baseline has been accurately measured (assuming the instrument is working properly).

It may be difficult to measure temperature in the desired manner because the end points of the line are elevated and the intervening terrain is much lower. In this case, elevate the end point temperature measuring devices as high as possible, and take a measurement 1 hour after sunrise and a second measurement 1 hour before sunset. The best conditions for measuring baselines are an overcast day with moderate winds to mix the air near the ground. As mentioned before, pressure measurements need be taken only at the end points to 0.01 inch of Hg.

A correction for humidity has not been mentioned because the errors owing to water vapor are almost always small in comparison with temperature errors at optical wavelengths. If desired, the humidity correction for average conditions may be added to the final result.

PART II.

MEASUREMENTS OF LARGE STRUCTURES

RATIOS. It was shown previously that refractive index errors limit the accuracy of DME. When temperature and pressure are measured properly, errors still occur because it is difficult or impossible to measure other than at the end points of the line. Further, refractive index measurements are both time consuming and expensive. This section discusses techniques for reducing refractive index errors in measurements of large structures by using ratios or reference lines.

A number of studies, made as a part of a research program in the most accurate use of DME, have resulted in the formulation of two important experimental rules:

1. Refractive index errors, resulting from end point measurements of temperature and pressure, tend to be the same for all lines measured from one point within a short period of time.

2. The ratios of observed distances, measured from one point within a short period of time, are constant.

For both rules, a short period of time is 30 minutes or less.

In figure 4, lines AB and AC are measured from a common point. Rule 1 states that if refractive index measurements are made at points A, B, and C within a short period, the errors in the measurements tend to be the same at all three points. If the true temperature along line AB is 20°C, but the mean of measurements made at A and B is 24°C (a condition typical of daytime), then the expected true temperature of line AC would be approximately 4°C cooler than the end point temperature measurements taken at A and C. Because 1°C is approximately equivalent to one-part-per-million of distance, both lengths will be in error by 4 ppm.

$$\begin{aligned} AB &= AB(1 \pm 4 \text{ ppm}) \\ AC &= AC(1 \pm 4 \text{ ppm}) \end{aligned} \tag{5}$$

If the two equations are divided, the errors are canceled, and the ratio will be more accurate than either of the two lengths from which it was derived. Much of the work on dams may be performed using ratios as observed quantities.

In many respects, ratios have properties similar to those of angles. In triangulation, the sum of the three angles of a triangle must equal 180°, and a knowledge of two angles permits calculation of the third. Similarly, the product of three ratios obtained from

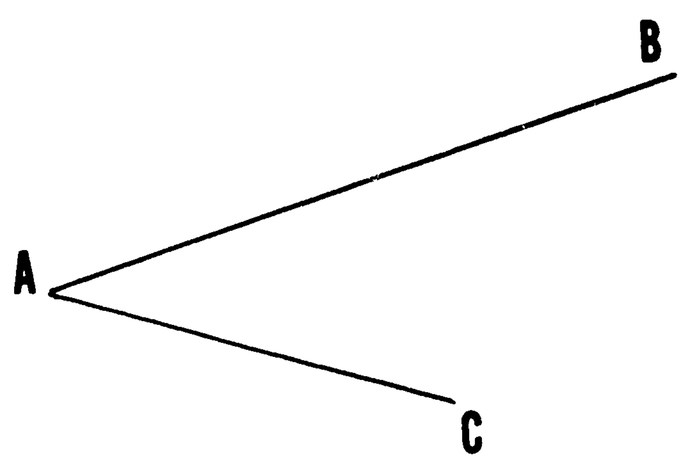


Figure 4. Ratio of Two Lines

a triangle must equal 1 and a knowledge of two ratios permits calculation of the third. In figure 5, the triangle shown has sides A, B, and C as measured from vertices 1, 2, and 3. The ratio measured from vertex 1 is A_1/B_1 using a counterclockwise convention (A_1/B_1 rather than B_1/A_1) with the subscript designating the vertex from which the ratio was measured. Two other ratios, B_2/C_2 and C_3/A_3 may also be measured. If the measurements are perfect, $A_1 = A_3$, $B_1 = B_2$, $C_2 = C_3$, and $A_1/B_1 \times B_2/C_2 \times C_3/A_3 = 1$. If the measurements are not perfect (the usual case), the degree to which the product failed to equal 1 is a measure of the precision of the measurements. If only two ratios were measured, the third may be calculated. For example, $A_1/B_1 = C_2/B_2 \times A_3/B_3$. Angles may be calculated directly from the ratios through use of a modified cos formula:

$$\cos \angle 1 = \tfrac{1}{2}\left[\frac{A_1}{B_1} + \frac{B_1}{A_1} - \left(\frac{C_3}{A_3} \times \frac{C_2}{B_2}\right)\right] \tag{6}$$

The use of ratios yields angles as a result, and the angles determined from the ratios are more accurate than those determined from the lengths alone because a ratio is more accurate than either of the lengths from which it is derived.

When the angles of a triangle do not sum to 180°, the triangle may be adjusted by taking one-third of the difference between 180° and the sum of the angles and by applying it as a correction to each angle. With ratios, a correction may be made to each ratio. Let E, F, and G be three ratios obtained from the vertices of a triangle. For a perfect set of measurements, $E \times F \times G = 1$. For an imperfect set of measurements, let $W = (E \times F \times G) - 1$. Form new ratios so that

$$E' = \left(\frac{E}{1 + \frac{W}{3}}\right) \qquad F' = \left(\frac{F}{1 + \frac{W}{3}}\right) \qquad G' = \left(\frac{G}{1 + \frac{W}{3}}\right) \tag{7}$$

Now $E' \times F' \times G' = 1$ and the ratios have been adjusted.

Thus far, we have been using <u>corrected ratios</u>; that is, ratios that have been formed from lines that were corrected by means of temperature and pressure measurements made at the time.

A second set of ratios can be obtained from the same measurements by using the data before the application of the refractive index corrections. These are the <u>observed ratios</u> and have been formed from lines that have had no temperature or pressure correction applied but have been corrected for the instrument and reflector constants. If, once again, the product of three observed ratios is

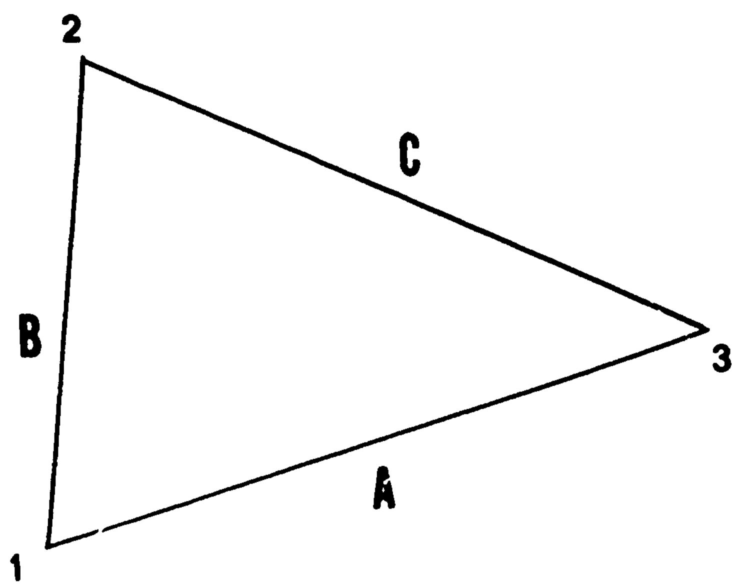


Figure 5. Ratios in a Triangle

taken, it will be seen to be close to 1, although no atmospheric correction has been applied. In most cases, the observed product will be closer to 1 than the corrected product. Referring again to figure 5, imagine three identical instruments set on the three vertices of the triangle. Line A is measured from both ends at the same time, then the same thing is done with line B, and finally line C. The product condition, $A_1/B_1 \times B_2/C_2 \times C_3/A_3 = 1$, will hold true because $A_1 = A_3$, $B_1 = B_2$, and $C_2 = C_3$ as a consequence of each line having been measured from both ends at the same time. The second rule given at the beginning of this section states that the three ratios will remain constant with time. If this is true, it means that A_1/B_1 may be measured one day and B_2/C_2 months later. The individual observed lengths change considerably with time, but the ratio of lines measured from the same point within a short period of time remains constant. Later this rule will be put to use in measurements of dams.

Let us assume for a moment that an instrument has been set upon a hilltop. In the valley below, two points have been selected that are equidistant from the hilltop stations and are at the same elevation. The observed distances to the two points would appear the same because the distances are equal and both lines pass through roughly the same atmosphere. A point is then selected that is the same distance from the hilltop station as the other points, but with a higher elevation. When the observed distances are recorded, the two lengths to the valley points are the same, but the observed length to the higher elevation point is shorter. This is because air density decreases with elevation, and the light traversing the higher line travels faster and returns sooner. The instrument then shows the distance to be shorter. Two lessons can be learned from this. The first lesson is that if the mean elevations of two lines measured from a point are the same, the ratio of the observed distances will be accurate. This is often the case with dams where the alignment markers on the crest of a dam are all within a few feet of the same elevation. In the case of the two valley lines, the ratio would be 1 because the lines are of equal length and elevation. This property of line ratios will be used later on. The second lesson is that when the elevations of the end points to which measurements are being made are different, the ratio of observed lengths is not the same as the ratio of the true lengths because the refractive indices along the two lines are different. Even though the observed ratio is not correct, it does not change with time. When differences in position, as a function of time, are more important than knowledge of the correct position itself, as in movements of dams, the observed ratio may often be used even when large differences in elevation occur. Furthermore, the observed ratio can be corrected by using a simple atmospheric model.

Imagine a day on which the sea level temperature was 20°C and the

sea level pressure was 760 mm (29.92 in.) of Hg. In figure 6, two lines are shown. Station A has an elevation of 322 feet; Station B, an elevation of 505 feet; and Station C, an elevation of 1,837 feet. The equation used in barometric altimetry will permit us to calculate the pressure at the midpoint of each line based on a sea level pressure of 29.92 inches of Hg. The equation used is $P_h = P_0(1 - 0.0000068754h)^{5.2561}$, where P_h is the barometric pressure at elevation h, and P_0 is the sea level pressure. Further, we may calculate a temperature for the line using a convenient lapse rate, 2.5°C per 1,000 feet of elevation is a common value. For line AB with a mean elevation of 413.5 feet, the pressure would be 29.476 inches of Hg and the temperature would be 19.0°C. For line BC with a mean elevation of 1,171 feet, the values would be 28.675 inches of Hg and 17.1°C, respectively. From these values the refractive index along each line may be calculated, and a correction applied to each observed distance. Then if the ratio of the two lines is taken, it will be the correct ratio because a refractive index correction has now been applied. To understand how this might be used in practice, observed distances for lines AB and BC are taken:

$$\begin{aligned} AB &= 12{,}064.182 \text{ feet} \\ BC &= 12{,}064.113 \text{ feet.} \end{aligned} \tag{8}$$

The observed ratio is equal to 1.0000057. After applying the refractive index corrections for each line based on the temperature and pressure values derived from an atmospheric model,

$$\begin{aligned} AB &= 12{,}064.221 \\ BC &= 12{,}064.221 \end{aligned} \tag{9}$$

the corrected ratio is 1.0000000. Note that the value of each line, 12,064.221, is probably not the correct length, but 1.0000000 is the correct ratio.

<u>MEASUREMENTS OF LARGE STRUCTURES</u>. Measurements of movements in large structures can be made very accurately, in two dimensions, using trilateration techniques. The work consists of two phases, the control network, and the structure itself.

<u>The Control Network</u>. In monitoring possible movements of structures, points on the structure must be related to points that have been selected for stability, usually at some distance from the structure itself. These will be called control points, and all movements of the structure will be related to one or more of them. It is important that the control points not move, and for this reason, they should be placed in geologically stable positions. They should also afford a good geometry for trilateration measurements. Good geometry, in turn, consists of measuring along the line where movement is expected. For example, if measurements of upstream or

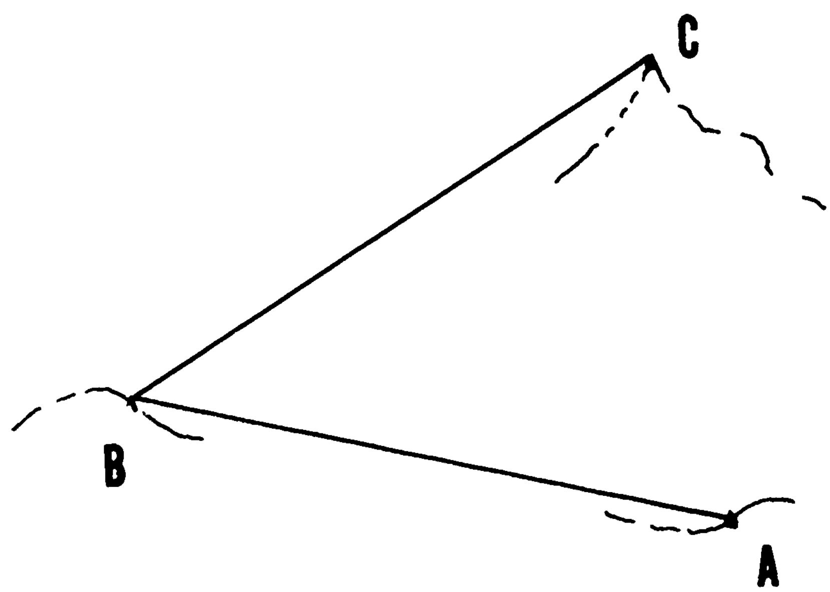


Figure 6. Stations with Different Elevations

downstream movements are required, the control point should be located either upstream or downstream. Further, the point should be at a sufficient distance from the structure so that the end points, as well as the center, can be monitored with good geometry. In figure 7, a dam is shown with both an upstream and downstream control monument. Geometrically, measurements from the upstream side of the dam will be poor; however, those from the downstream side will be much stronger. If movement in two dimensions is desired, a point off the end of the dam should also be chosen as shown in figure 7. For best results, the angle of intersection (θ) should be 90°, but in any case, it should be between 30° and 150°.

A final criteria for the selection of control monuments is intervisibility. Because the control figure may also provide a means of correcting for refractive index, the points selected for control at the ends of the dam must be visible from the upstream and/or downstream points.

In trilateration, lengths to an unknown station from each of two control points will give the position of the unknown station in two dimensions. Measurements from three control stations will give three positions of the unknown station, which may be used as a check of survey accuracy. Figure 8 shows an ideal control figure for the measurement of a dam. In the control figure A, B, C, and D are monuments. All are intervisible. Point D, in particular, has been selected to be visible from the other three. Point P is an unknown station on the dam and is measured from control points A, B, and C. Positions of P are calculated from measurements A and B, from measurements B and C, and from measurements A and C.

When measurements are made of lines exceeding 2,000 feet in length, a major source of error is the inability to determine accurately the refractive index along the line. An error in temperature of 1°C or in pressure of 0.1 inches of Hg will cause an error in length of one-part-per-million. These errors may be minimized by considering the ratio of two lines that have been measured within 30 minutes of each other. The errors of each line tend to be the same so that taking a ratio greatly reduces the magnitude of the error. This may be shown by again referring to figure 8. Point D has been selected as a reference point. Its position was chosen to place it in stable ground, to be visible from the other control points, and so that the lines to it from the other control points would pass through similar atmospheric conditions to those from the control points to unknown positions on the dam.

The first time the dam is visited for the purpose of making trilateration measurements, conventional techniques are used to determine the size and shape of the control figure. This requires careful temperature and pressure measurements at each end of each

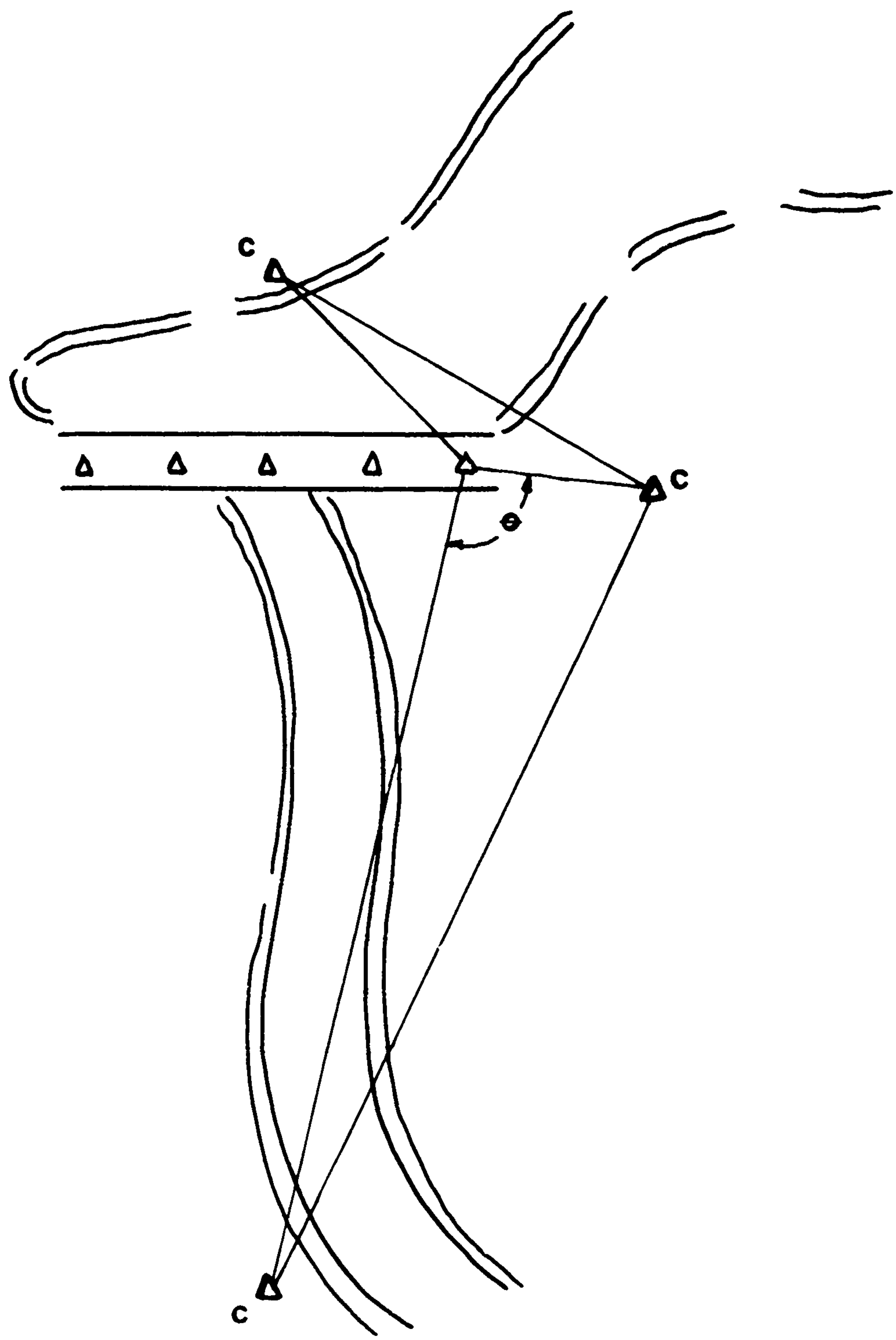


Figure 7. Control Monuments for a Dam

line that is measured. All control points should be occupied with the DME, and measurements should be made to all visible control points. From any single station the lines should be measured as rapidly as possible, perferably within one-half hour. The distance to the reference monument D should be read twice.

A typical set of measurements might proceed by first occupying control point A (figure 8). Measurements would, in turn, be made to points D, B, C, and again D. The object of measuring twice to D is to check for drift in the instrument or changes in the atmosphere.

When all the control points have been occupied and all the possible lines measured, the figure may be reduced to a series of triangles. Before the angles can be calculated, each line must be reduced to a chord distance on the spheroid. This reduction is necessary to apply geometric checks to figures or to calculated positions from several sets of data that agree with each other. An earth radius for each dam must be selected from the table in the appendix. With this earth radius, the equation for the spheroid reduction is

$$Ds = R\sqrt{\frac{[D_1 - (e_2 - e_1)][D_1 + (e_2 - e_1)]}{(R + e_2)(R + e_1)}} \tag{10}$$

where

Ds = Spheroid chord distance
R = Earth radius
D_1 = Observed distance from the DME
e_1 = Elevation + H.I. of the instrument
e_2 = Elevation + H.I. of the reflector

When reducing the data to the spheroid, use the observed distance before the refractive index correction. If your instrument has a dial-in refraction correction, set the dial to zero. After obtaining the observed spheroid distance, apply the refraction correction. Record both the observed spheroid and the corrected spheroid distances. The corrected spheroid distances are used to form corrected ratios, which in turn are used to calculate the angles of triangles in the control figure. The most accurate line should be used for scale. The equations for calculating angles from ratios are found in the section on ratios, and in the appendix. The observed spheroid distances are retained for future use. After the control figure has once been measured using temperature and pressure correction, possible movements of control points may be monitored by comparing observed ratios. If these do not change, the control figure has not changed. This technique avoids the requirement for future

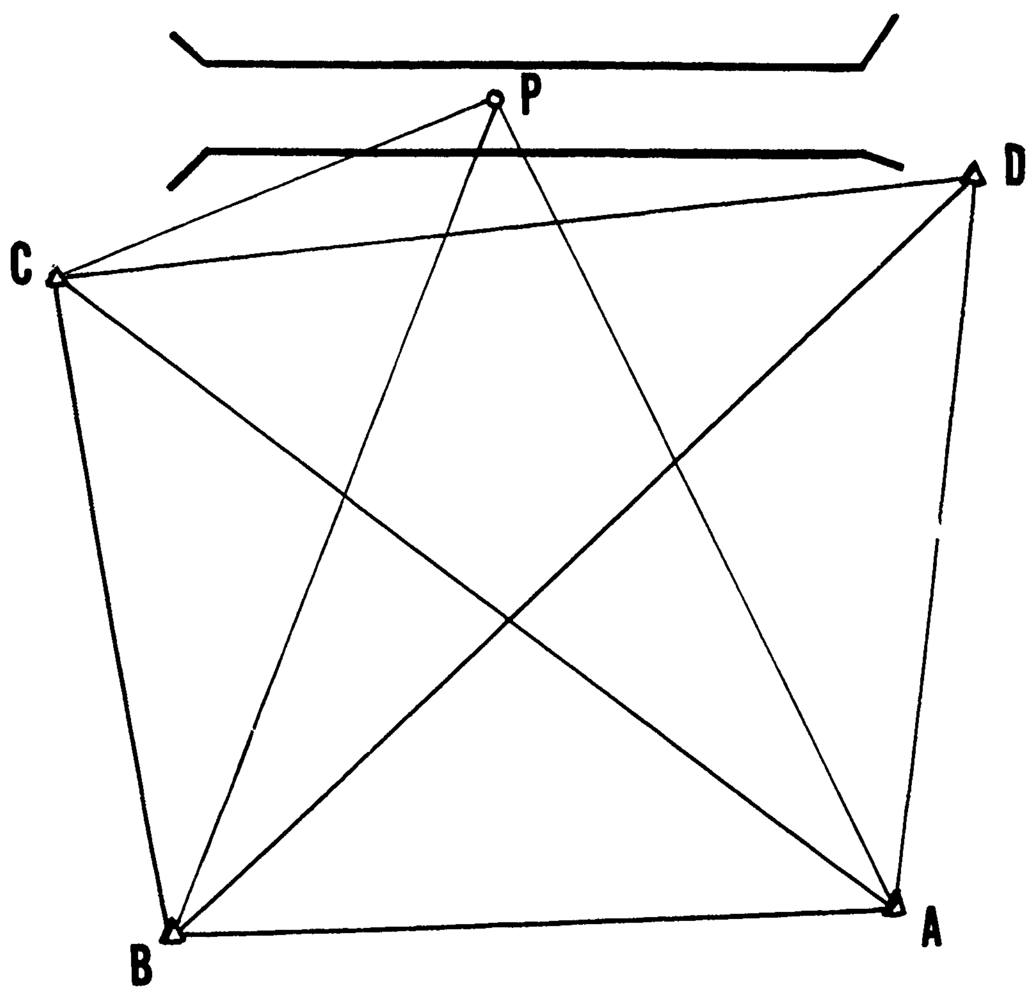


Figure 8. An Ideal Control Figure

refractive index measurements, while actually improving the accuracy and sensitivity of trilateration.

Points on the Dam. When positions have been established for the monuments in the control figure, the length from the control monument to the reference monument may be used to monitor the refractive index for lines to points on the dam. This may be done by assuming the distance to the reference to be correct. This correct distance to the reference monument (R corr) together with the observed distance (R obs) provide a correction factor for refractive index that may be applied to other lines from the control monument to points on the dam. The corrected distance would be

$$P \text{ corr} = \frac{R \text{ corr}}{R \text{ obs}} \times P \text{ obs}. \qquad (11)$$

In the next section, all of the techniques are illustrated by using a fictitious dam.

EXAMINATION OF A FICTITIOUS DAM. The concrete dam represented in figure 9 is located at 40° latitude. Control pedestals have been set at points C1, C2, C3, and C4. Alignment markers A1 through A6 have been set along the crest of the dam, and markers T1 and T2 have been set at the toe of the dam. Elevations have been measured to obtain the following list:

	Elevations (ft above sea level)
A1	1,347.52
A2	1,347.50
A3	1,347.46
A4	1,347.51
A5	1,347.48
A6	1,349.23
C1	1,377.66
C2	1,355.89
C3	1,521.33
C4	1,711.08
T1	1,081.44
T2	1,080.69

The control monuments were occupied with DME, and measurements were made to all visible monuments on the dam and to all other control points. On 2 February, the following measurements were made from C3.

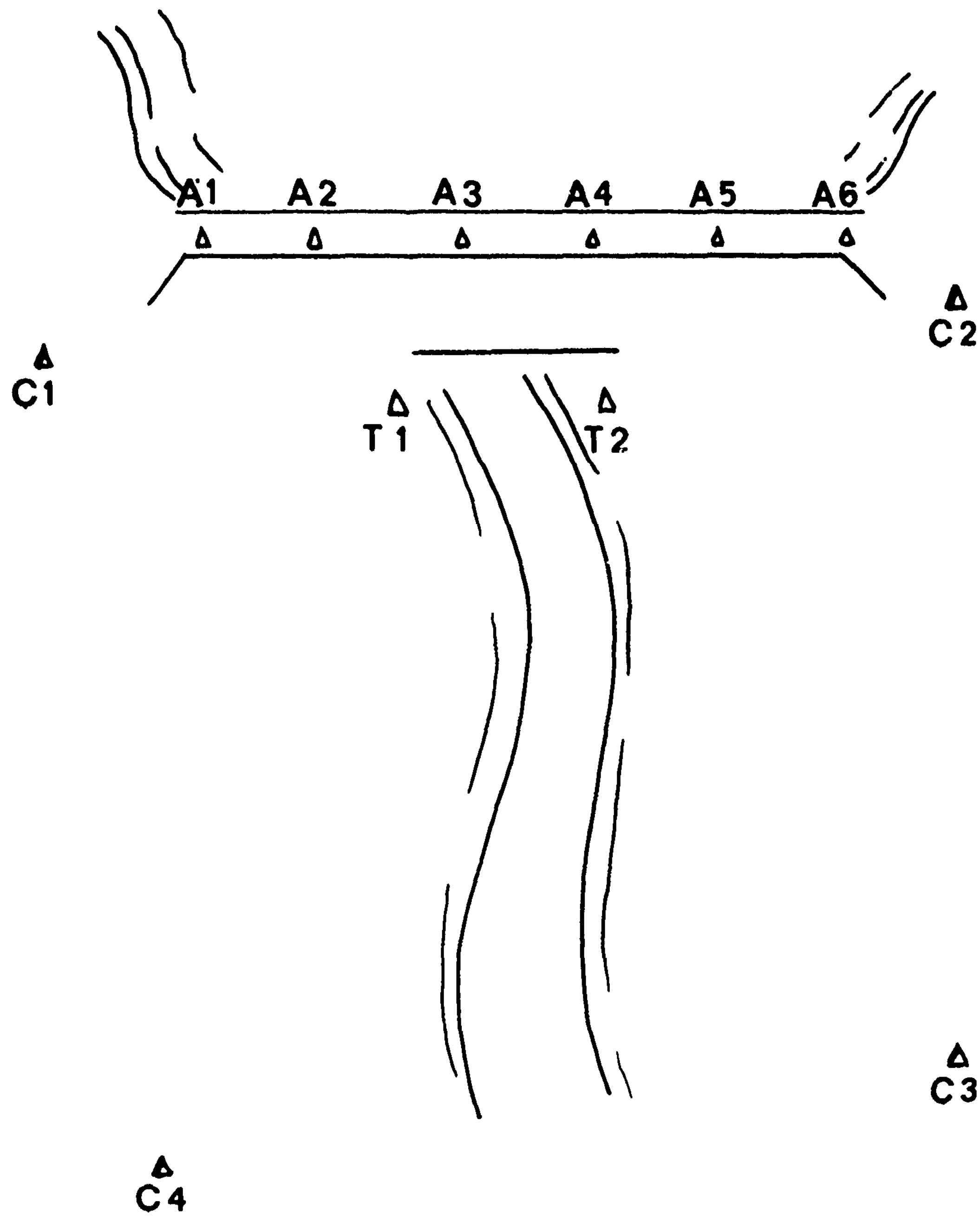


Figure 9. A Fictitious Dam

Table 6. Measurements from C3, 2 Feb.

MEAS	TO	TIME	OBSERVED DISTANCE (feet)	MEAN TEMPERATURE (°C)	MEAN PRESSURE (in. Hg)
1	C1	0930	3,546.933	16.4	28.26
2	C4	0935	3,100.500	16.4	28.09
3	C2	0940	2,309.017	17.0	28.27
4	C1	0945	3,546.930	16.7	28.26
5	C1	1025	3,546.927		
6	A1	1035	3,176.645		
7	A2	1045	3,032.982		
8	C1	1050	3,546.925		
9	A3	1100	2,896.083		
10	A4	1115	2,769.433		
11	C1	1120	3,546.929		
12	A5	1130	2,646.391		
13	A6	1145	2,535.926		
14	C1	1150	3,546.928		
15	C1	1300	3,546.925		
16	T1	1305	2,863.798		
17	T2	1315	2,742.852		
18	C1	1320	3,546.922		

The monument C1 has been taken as the reference monument for this study. The measurements begin with the control figure. Temperature and pressure measurements are made only on the control lines and only the first time a study is made at the particular dam. The next time the dam is visited, perhaps 6 or 12 months later, it will not be necessary to measure refractive index. Possible movement in the control figure may be checked at that time by a comparison of ratios of observed distances.

Measurements are made that same afternoon from C1.

Table 7. Measurements from C1, 2 Feb.

MEAS	TO	TIME	OBSERVED DISTANCE (feet)	MEAN TEMPERATURE (°C)	MEAN PRESSURE (in. Hg)
19	C2	1400	1,857.665	19.0	28.28
20	C3	1405	3,546.923	18.8	28.20
21	C4	1410	3,246.133	18.5	28.09
22	C2	1415	1,857.661	18.8	28.28
23	A6	1420	1,557.586		
24	A5	1430	1,358.043		
25	C2	1440	1,857.665		

Table 7 (con't)

MEAS	TO	TIME	OBSERVED DISTANCE (feet)	MEAN TEMPERATURE (°C)	MEAN PRESSURE (in. Hg)
26	A4	1450	1,158.663		
27	A3	1505	959.505		
28	C2	1510	1,857.665		
29	A2	1520	760.838		
30	A1	1530	563.045		
31	C2	1540	1,857.663		

The monuments at the toe of the dam are not visible from C1. The line from C1 to C2 is used for a reference for measurements made from C1. This line is chosen as a reference because it closely duplicates the atmospheric conditions along the lines to A1 through A6.

A week later monument C4 was occupied.

Table 8. Measurements from C4, 9 Feb.

MEAS	TO	TIME	OBSERVED DISTANCE (feet)	MEAN TEMPERATURE (°C)	MEAN PRESSURE (in. Hg)
32	C1	0835	3,246.199	6.1	28.85
33	C2	0840	3,734.494	6.1	28.87
34	C3	0845	3,100.550	5.8	28.78
35	C1	0850	3,246.198	6.2	28.85
36	C1	0900	3,246.198	6.2	28.84
37	A1	0905	3,384.466		
38	A2	0915	3,421.825		
39	A3	0925	3,470.325		
40	C1	0930	3,246.188		
41	A4	0940	3,529.493		
42	A5	0945	3,598.824		
43	A6	0955	3,677.581		
44	C1	1000	3,246.181		
45	T1	1010	3,219.499		
46	T2	1020	3,240.423		
47	C1	1025	3,246.179		

Then finally measurements were made from monument C2.

Table 9. Measurements from C2, 9 Feb.

MEAS	TO	TIME	OBSERVED DISTANCE (feet)	MEAN TEMPERATURE (°C)	MEAN PRESSURE (in. Hg)
48	C1	1230	1,857.594	8.1	29.04
49	C4	1235	3,734.491	7.6	28.87
50	C3	1240	2,309.052	7.8	28.97
51	C1	1245	1,857.694	8.3	29.04
52	A1	1250	1,306.266		
53	A2	1300	1,106.790		
54	A3	1310	907.650		
55	C1	1315	1,857.692		
56	A4	1320	708.919		
57	A5	1330	511.246		
58	A6	1335	316.414		
59	C1	1345	1,857.690		
60	T1	1355	1,122.618		
61	T2	1400	983.806		
62	C1	1410	1,857.688		

The first step in the data reduction is to bring all the lines down to the spheroid. The values in table 10 are derived by using a latitude of 42° and by using the table and equation in the appendix.

Table 10. Corrected Line Lengths

MEAS	C3 to	OBS SPHEROID DIST (ft)	CORR SPHEROID DIST (ft)
1	C1	3,543.777	3,543.820*
2	C4	3,094.449	3,094.492*
3	C2	2,302.925	2,302.954*
4	C1	3,543.774	3,543.818*
5	C1	3,543.771	
6	A1	3,171.669	3,171.711
7	A2	3,027.789	3,027.829
8	C1	3,543.769	
9	A3	2,890.661	2,890.699
10	A4	2,761.161	2,761.197
11	C1	3,543.773	
12	A5	2,640.493	2,640.526
13	A6	2,529.906	2,529.938
14	C1	3,543.772	
15	C1	3,543.769	
16	T1	2,829.636	2,829.671
17	T2	2,707.058	2,707.092
18	C1	3,543.766	

Table 10 (con't)

MEAS	C1 to	OBS SPHEROID DIST (ft)	CORR SPHEROID DIST (ft)
19	C2	1,857.416	1,857.443*
20	C3	3,543.767	3,543.820*
21	C4	3.228.726	3,228.777*
22	C2	1,857.412	1,857.439*
23	A6	1,557.225	1,557.247
24	A5	1,357.619	1,357.638
25	C2	1,857.416	
26	A4	1,158.195	1,158.210
27	A3	958.967	958.979
28	C2	1,857.416	
29	A2	760.190	760.200
30	A1	562.201	562.209
31	C2	1,857.414	
	C4 to		
32	C1	3,228.792	3,228.781*
33	C2	3,717.292	3,717.279*
34	C3	3,094.499	3,094.490*
35	C1	3,228.791	3,228.781*
36	C1	3,228.791	
37	A1	3,364.637	3,364.624
38	A2	3,402.206	3,402.195
39	A3	3,450.970	3,450.962
40	C1	3,228.781	
41	A4	3,510.461	3,510.457
42	A5	3,580.147	3,580.145
43	A6	3,659.468	3,659.468
44	C1	3,228.774	
45	T1	3,157.115	3,157.118
46	T2	3,178.298	3,178.305
47	C1	3,228.766	
	C2 to		
48	C1	1,857.445	1,857.439*
49	C4	3,717.289	3,717.281*
50	C3	2,302.959	2,302.953*
51	C1	1,857.445	1,857.439*
52	A1	1,306.155	1,306.152
53	A2	1,106.687	1,106.685
54	A3	907.552	907.550
55	C1	1,857.443	
56	A4	708.824	708.823

Table 10 (con't)

MEAS	C2 to	OBS SPHEROID DIST (ft)	CORR SPHEROID DIST (ft)
57	A5	511.144	511.143
58	A6	316.323	316.323
59	C1	1,857.441	
60	T1	1,088.490	1,088.488
61	T2	944.476	944.475
62	C1	1,857.439	

*Length corrected from temperature and pressure measurements.

The control lines are then corrected for refractive index. Again, this is the only time these corrections need be made. The control lines are marked with an asterisk.

The next step is to fit the control figure into a coordinate system. This might be the local State Coordinate system or it might be one used just for this project.

We have selected a system for just this dam and have arbitrarily selected C1 as a starting point with coordinates

$$\text{C1} \quad X = 10{,}000 \quad Y = 10{,}000 \tag{12}$$

The selection of a second point for the system will give the control figure both orientation and scale. In the case of our fictitious dam, we will choose the line from C1 to C2. Although we might prefer one of the longer lines, we believe C1 - C2 to have been more accurately measured. Table 11 is a summary of the corrected lengths of the control lines.

Table 11. Control Line Lengths

MEAS	LINE	CORR SPHEROID DIST (ft)
19	C1 - C2	1,857.443
20	C1 - C3	3,543.820
21	C1 - C4	3,228.777
22	C1 - C2	1,857.439
48	C2 - C1	1,857.439
49	C2 - C4	3,717.281
50	C2 - C3	2,302.953
51	C2 - C1	1,857.439

Table 11 (con't)

MEAS	LINE	CORR SPHEROID DIST (ft)
1	C3 - C1	3,543.820
2	C3 - C4	3,094.492
3	C3 - C2	2,302.954
4	C3 - C1	3,543.818
32	C4 - C1	3,228.781
33	C4 - C2	3,717.279
34	C4 - C3	3,094.490
35	C4 - C1	3,228.781

We have selected C1 - C2 for scale purposes, and the second point in the coordinate system is C2 at a distance of 1,857.440 feet (the mean of the four corrected measurements). The coordinates for the second points are those which are in approximately the proper direction from the first and give a distance of 1,857.440 feet.

$$\text{C2} \quad X = 11{,}843.620 \qquad Y = 9{,}773.841 \tag{13}$$

Turning now to the measurements made from C1, we find that the first measurement of C1 - C2 was slightly long (#19) and that the second was slightly short (#22) compared with the mean distance of 1,857.440. We may use the knowledge of this small difference in length to apply a correction to measurements #20 and #21 by assuming that the difference occurs because of changes in the atmosphere as the measurements proceded. Thus, the following table might be constructed:

Table 12. Correction for Refraction

MEAS #	C1 - C2 DIST	CORRECTION
19 (measured)	1,857.4430	1,857.4400/1,857.4430 = 0.999998385
20 (assumed)	1,857.4417	1,857.4400/1,857.4417 = 0.999999085
21 (assumed	1,857.4403	1,857.4400/1,857.4403 = 0.999999839
22 (measured)	1,857.4390	1,857.4400/1,857.4390 = 1.000000538

true length = 1,857.4400

The table shows what the observed value of the reference line might have been expected to be during the time the measurements to C3 and C4 were made. The correction, which is the ratio of the correct distance divided by what would have been the observed distance at the time, may then be applied to measurements #20 and #21 with the following result:

#20	C1 - C3	3,543.817
#21	C1 - C4	3,228.776

Similarly, measurements #49 and #50 might be corrected to

#49	C2 - C4	3,717.283
#50	C2 - C3	2,302.954

These four measurements, together with the positions of C1 and C2, may now be used to determine the positions of C3 and C4 using the appropriate equation in the appendix.

$$\begin{array}{ll} C3 & X = 12{,}609.103 \\ & Y = 7{,}601.830 \\ \\ C4 & X = 9{,}622.072 \\ & Y = 6{,}793.419 \end{array} \tag{14}$$

The distance C3 - C4 may now be used to check the accuracy of the two positions. The calculated difference in position is 3,094.492 feet, and the mean measured length is 3,094.491 feet.

It should be obvious at this point that there are other ways to obtain the positions of the control monuments, and in a practical situation, it might be desirable to adjust the result. Although adjustment is beyond the scope of this manual, the single triangle adjustment given previously may be used to determine the individual angles of a double-braced quadrilateral. The figure may then be adjusted as if it were one in which all the angles had been measured with one side acting as a baseline.

At this point, the control figure has been fixed with the position of each control monument assigned a position. Three lines in the figure have become reference lines to be used with measurements to the dam from the four control monuments. The lengths of the reference lines are obtained from differences in position.

MONUMENT	REF LINE	LENGTH
C1	C1 - C2	1,857.440
C2	C1 - C2	1,857.440
C3	C1 - C3	3,543.817
C4	C1 - C4	3,228.776

The reference lengths will be used to correct measurement to points on the dam for refractive index. Let us examine measurements 36 through 44 made from C4 and taken from table 10 (see table 13). For measurement 36 the observed length of the reference line is longer than the true length. The reason is the atmosphere. If the observed

Table 13. Changes of Correction Factor with Time

MEAS #	TO	OBS SPHEROID DISTANCE	CORRECTION FACTOR	CORR SPHEROID DISTANCE
36	C1	3,228.791	(0.99999535)	
37	A1	3,364.637	0.99999612	3,364.624
38	A2	3,402.206	0.99999690	3,402.195
39	A3	3,450.970	0.99999767	3,450.962
40	C1	3,228.781	(0.99999845)	
41	A4	3,510.461	0.99999899	3,510.457
42	A5	3,580.147	0.99999953	3,580.145
43	A6	3,659.468	1.00000007	3,659.468
44	C1	3,228.774	(1.00000062)	

TRUE REFERENCE LENGTH 3,228.776

distance were multiplied by a factor of 3,228.776/3,228.791 = 0.99999535, it would equal the true length. This is the atmospheric correction factor for that time. For measurement 40 a new factor may be calculated. The difference is due to gradual changes in atmospheric conditions. We may assume that this change has occurred in a linear fashion over a short period of time, and a correction factor should be calculated for measurements 37, 38, and 39. The correction factors are then applied to the observed distances to obtain the corrected distances in the last column of the table. These are distances that have been corrected for refractive index by using a reference line. The values obtained this way are quite accurate as long as the reference line has approximately the same elevation. Although this is true for the monuments along the crest, it is not true for the monuments at the toe of the dam, which are at a lower elevation. There are two calculations, either of which may be chosen, to determine these lines; (1) perform the same calculation as was done with the crest markers, or (2) use the reference line together with an atmospheric model. In the first calculation, the position determined for the toe monuments will be slightly in error, but the error will be the same each time the measurements are made and thus, for detecting movements, will be adequate. In the second calculation, accurate positions are required; the lengths may be determined (see Ratios section) by using the equation in the appendix. Results using both methods are listed below for measurements 45 and 46. Number 1 uses the reference line only, and number 2 uses the reference line together with an atmospheric model to correct for difference in elevation. The differencesin these two examples are small; larger differences in elevation and longer lines would give greater differences. The difference in mean elevation between lines C4 - C1 and C4 - T1 is only about 150 feet, and the difference in the mean atmospheric pressure would be approximately 0.150 inches of Hg, or 1.5 ppm of distance.

MEAS #	TO	OBS SPHEROID DISTANCE	CORR SPHEROID DISTANCE
44	C1	3,228.774	
45	T1	3,157.118	(1) 3,157.123
			(2) 3,157.118
46	T2	3,178.302	(1) 3,178.309
			(2) 3,178.305
47	C1	3,228.766	

Applying these techniques to the rest of the observed distances, we finally obtain the values in table 10. The corrected spheroid distances are in the last column of the table. Those followed by an asterisk have been corrected by temperature and pressure measurements and those not followed by an asterisk have been corrected by reference line corrections.

When corrected spheroid distances have been calculated for all lines, positions may be determined for the six alignment markers on the dam and for the two markers at the toe. A position may be calculated from the two control monument positions and from the two lengths from the control monuments to the unknown station. When two lines have been measured, one position may be determined. If the unknown station has been measured from three control monuments, three positions may be determined; if it has been measured from four monument, six positions may be determined.

In table 14, the positions of the six alignment markers and the two toe markers are given. Six positions are given for the alignment

Table 14. Alignment and Toe Marker Positions

STATION	X	LINES	Y
A1	10,561.688	C1 - C2	10,024.206
	" .688	C1 - C3	" .193
	" .689	C1 - C4	" .180
	" .685	C2 - C3	" .190
	" .683	C2 - C4	" .181
	" .677	C3 - C4	" .183
A2	10,760.200	C1 - C2	9,999.568
	" .200	C1 - C3	" .601
	" .200	C1 - C4	" .600
	" .208	C2 - C3	" .607
	" .206	C2 - C4	" .598
	" .199	C3 - C4	" .600

Table 14 (con't)

STATION	X	LINES	Y
A3	10,958.654	C1 - C2	9,975.004
	" .654	C1 - C3	" .047
	" .654	C1 - C4	" .036
	" .655	C2 - C3	" .048
	" .653	C2 - C4	" .037
	" .644	C3 - C4	" .040
A4	11,157.149	C1 - C2	9,950.442
	" .150	C1 - C3	" .455
	" .150	C1 - C4	" .450
	" .153	C2 - C3	" .457
	" .151	C2 - C4	" .449
	" .145	C3 - C4	" .452
A5	11,355.613	C1 - C2	9,925.883
	" .612	C1 - C3	" .865
	" .613	C1 - C4	" .874
	" .607	C2 - C3	" .862
	" .611	C2 - C4	" .875
	" .621	C3 - C4	" .870
A6	11,554.117	C1 - C2	9,901.309
	" .116	C1 - C3	" .306
	" .116	C1 - C4	" .298
	" .115	C2 - C3	" .305
	" .113	C2 - C4	" .300
	" .109	C3 - C4	" .302
T1	10,755.	C1 - C2	9,740.
	" .	C1 - C3	" .
	" .	C1 - C4	" .
	" .658	C2 - C3	" .002
	" .658	C2 - C4	" .006
	" .661	C3 - C4	" .005
T2	10,901.	C1 - C2	9,702.
	" .	C1 - C3	" .
	" .	C1 - C4	" .
	" .830	C2 - C3	" .679
	" .829	C2 - C4	" .688
	" .837	C3 - C4	" .684

markers (measurements from four stations), and three positions are given for the toe markers (measurements from three stations). Some measurements will be better than others because of geometry. When

an answer is suspect, it may be because of geometry or it may be because of a bad line. A bad line, however, will give a bad result each time it is used and will usually affect more than one answer. For example, the first Y value listed for A2 is probably caused by bad geometry. If the measurement from C1 was bad, the first three values would all show poor agreement. If line C2 was bad, values 1, 4, and 5 would give poor agreement.

The positions listed in the table may now be used to determine alignment. Any desired positions may serve as end points. In table 15, the second values of A1 and A6 serve as end points. Note that T1 and T2 may also be included in the alignment. Including T1 and T2 helps to monitor tilt in the dam. Alignment done from positions is not affected by curved dams, by dog legs, or by differences in elevations.

Table 15. Alignment

MONUMENT	DIST. FROM A1	DIST. OFF LINE
A2	200.029	-0.011
A3	399.997	+0.008
A4	600.010	-0.005
A5	799.990	-0.021
T1	344.077	-258.201
T2	468.047	-277.278

+ = UPSTREAM - = DOWNSTREAM

APPENDIX. EQUATIONS FOR USE IN THE REDUCTION OF DATA TAKEN AT DAMS

EARTH RADIUS FOR LINES OF 45° AZIMUTH

CLARK 1866 SPHEROID

LATITUDE	EARTH RADIUS FEET	EARTH RADIUS METERS
25°	20,880,000	6,364,200
26°	20,882,000	6,364,800
27°	20,883,000	6,365,400
28°	20,886,000	6,366,000
29°	20,888,000	6,366,600
30°	20,890,000	6,367,300
31°	20,892,000	6,367,900
32°	20,894,000	6,368,600
33°	20,896,000	6,369,300
34°	20,899,000	6,370,000
35°	20,901,000	6,370,700
36°	20,903,000	6,371,400
37°	20,906,000	6,372,100
38°	20,908,000	6,372,900
39°	20,910,000	6,373,600
40°	20,913,000	6,374,300
41°	20,915,000	6,375,100
42°	20,918,000	6,375,800
43°	20,920,000	6,376,600
44°	20,923,000	6,377,300
45°	20,925,000	6,378,100
46°	20,928,000	6,378,900
47°	20,930,000	6,379,600
48°	20,933,000	6,380,400
49°	20,935,000	6,381,100
50°	20,938,000	6,381,900

$$R = \frac{20784235}{1 - 0.010137\sin^2\phi - 0.006769\sin^2\alpha(1-\sin^2\phi)}$$

R = Earth radius in feet
ϕ = Latitude of line
α = Azimuth of line

REDUCTION OF A LINE TO A CHORD DISTANCE ON THE SPHEROID

$$D_c = R\sqrt{\frac{[D_1-(E_2-E_1)][D_1+(E_2-E_1)]}{(R+E_1)(R+E_2)}}$$

D_c = Spheroid Chord Distance
D_1 = Slope Distance
R = Earth Radius
E_1 = Elevation + H.I. (DME)
E_2 = Elevation + H.I. (Reflector)

REFRACTIVE INDEX OF THE AIR

A. Index of modulated light of wavelength λ at 0°C and 760 mm Hg.

$$n_g = 1 + \left(287.604 + \frac{4.8864}{\lambda^2} + \frac{0.068}{\lambda^4}\right)10^{-6}$$

n_g = Index of refraction at standard conditions*
λ = Wavelength of light used

*Modulated light only (group index)

B. Index at other temperatures and pressures

$$n_a = 1 + \left(\frac{n_g - 1}{1+\frac{T}{273.2}}\right)\left(\frac{P}{760}\right) - \left(\frac{5.5e}{1+\frac{T}{273.2}}\right)\cdot 10^{-8}$$

n_a = Index of refraction at desired temperature and pressure
n_g = Index of standard conditions
T = Temperature in C°
P = Pressure in mm Hg
e = Water vapor pressure in mm Hg (usually made zero)

C. Water Vapor Pressure (e) at 50% relative humidity

TEMPERATURE (°C)	WATER VAPOR PRESSURE (mm Hg at 50% R.H.)
-15	0.7
-10	1.1
-5	1.6
0	2.3
+5	3.2
+10	4.6
+15	6.4
+20	8.8
+25	11.9
+30	15.9
+35	21.1
+40	27.7

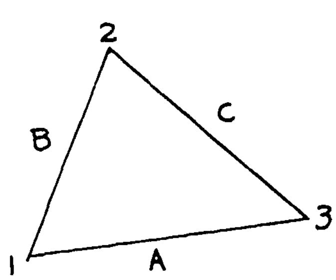
Water vapor correction will not exceed 1 part per million except under extremes of heat and humidity.

D. Refractive Index from a Model

$$n_a = 1 + \left(\frac{n_g - 1}{1 + \frac{T_o - Gh}{273.2}} \right) \left[\frac{P_o (1 - 0.0000068754\, h)^{5.2561}}{760} \right]$$

n_a = Index of refraction at model temperature and pressure
n_g = Index of refraction at standard conditions
T_o = Model temperature at reference height in C°
G = Temperature gradient in C°/ft
h = Height at which model index is desired, in ft, above reference height
Po = Model pressure at reference height in mm of Hg

COMPUTATIONS OF ANGLES FROM RATIOS



$$\cos\angle 1 = \frac{1}{2}\left[\frac{A_1}{B_1} + \frac{B_1}{A_1} - \left(\frac{C_3}{A_3} \times \frac{C_2}{B_2}\right)\right]$$

$$\cos\angle 2 = \frac{1}{2}\left[\frac{B_2}{C_2} + \frac{C_2}{B_2} - \left(\frac{A_1}{B_1} \times \frac{A_3}{C_3}\right)\right]$$

$$\cos\angle 3 = \frac{1}{2}\left[\frac{C_3}{A_3} + \frac{A_3}{C_3} - \left(\frac{B_2}{C_2} \times \frac{B_1}{A_1}\right)\right]$$

ADJUSTMENT OF THE THREE RATIOS OF A TRIANGLE

$$W = (E \times F \times G) - 1$$

$$E_1 = \frac{E}{1+\frac{W}{3}} \qquad F_1 = \frac{F}{1+\frac{W}{3}} \qquad G_1 = \frac{G}{1+\frac{W}{3}}$$

E, F, G = Unadjusted ratios
E_1, F_1, G_1 = Adjusted ratios

If $E_1 \times F_1 \times G_1 \neq 1.0000000$, Repeat adjustment letting

$E_1 = E$
$F_1 = F$
$G_1 = G$

INTERSECTION OF TWO LENGTHS FROM KNOWN POSITIONS

$$X_3 = X_1 + l_1 \sin(\theta - \phi)$$

$$Y_3 = Y_1 + l_1 \cos(\theta - \phi)$$

X_1, Y_1 = Coordinates of first known position
X_2, Y_2 = Coordinates of second known position
X_3, Y_3 = Coordinates of unknown position
l_1 = Distance from X_1, Y_1 to unknown position
l_2 = Distance from X_2, Y_1 to unknown position
l_0 = Distance between known positions

$$l_0 = \sqrt{(X_1 - X_2)^2 + (Y_1 - Y_2)^2}$$

$$\phi = \cos^{-1}\left(\frac{l_0^2 + l_1^2 - l_2^2}{2 l_0 l_1}\right)$$

$$\theta = \tan^{-1}\left(\frac{X_2 - X_1}{Y_2 - Y_1}\right) - 180^\circ$$

Two positions are obtained. The second position is found by interchanging X_1 and X_2; Y_1 and Y_2; l_1 and l_2 and repeating the calculation.

ALIGNMENT

$$d = l \sin(90^\circ - \theta)$$

$$\Delta = l \cos(90^\circ - \theta)$$

d = Distance along alignment line of unknown point

Δ = Distance perpendicular to alignment line of unknown point (misalignment)

X_1, Y_1 = Position of one end of alignment line

X_2, Y_2 = Position of the other end of the line

X_3, Y_3 = Position of unknown point

$$\theta = \left(\tan^{-1} \frac{Y_2 - Y_1}{X_2 - X_1}\right) - \left(\tan^{-1} \frac{Y_3 - Y_1}{X_3 - X_1}\right)$$

$$l = \sqrt{(X_3 - X_1)^2 + (Y_3 - Y_1)^2}$$